An Interview with Elizabeth Meckes, Part I

Dr. Elizabeth Meckes is an assistant professor of mathematics at Case Western Reserve University. She is a fellow of the American Institute of Mathematics and received her doctoral degree from Stanford under the direction of Persi Diaconis.

Ken: Hi Dr. Meckes, thank you so much for doing this interview for the Girls' Angle Bulletin. My first question for you is: What is your favorite kind of mathematics?

Elizabeth: This is a hard question to answer, so I'll answer it in a couple of ways. Firstly, if you had to classify me by area, you would call me a probabilist (someone who studies probability theory), and moreover, put me on the more "analytical" end, which means I use a lot of ideas and techniques from the field called "real analysis". One of the things I like about probability theory is that there are a lot of rather counter-intuitive results either about or using probability. I don't just like counter-intuitive results because I'm ornery; they can be a dramatic demonstration of the value of mathematics. Intuition can be wrong, and it can also break down and not tell you anything about a question you care about. Math is a way to move past intuition, and you know it's really done something for you when you come up with a result that you wouldn't have otherwise guessed was true.

To answer a different interpretation of the question, my favorite kind of math at any given moment is usually whatever I'm working on (broadly construed; it's not as though it's interesting every minute). I find that it's hard to be really interested in something until I've delved pretty deeply into it; real appreciation comes after understanding, which comes after a lot of hard work. This is a little unfortunate from one perspective, because it means you can't tell how interesting something will be to you without a lot of initial investment of time and energy, but it also means that you can be surprised by where you find interesting math.

Ken: How did you get interested in probability?

Elizabeth: Through my Ph.D. advisor, Persi Diaconis. I went to graduate school having almost no idea what I wanted to do, although I knew I was more inclined toward analytic (as opposed to algebraic) areas. The usual system to find a thesis advisor is to start by doing reading courses with several professors, sort of like a trial period. I did one with Persi, and enjoyed it. He seemed like someone I'd enjoy talking to once a week for a few years, so I asked him to be my advisor. I strongly advocate this as a way to choose an advisor: you can get interested in almost anything (and you can change areas later, although it's not easy), but it's important to have a good relationship with your advisor. If you dread those weekly meetings, graduate school will feel like a very unpleasant eternity. I could have done a lot of different things with Persi (he has very broad interests); I got into the part of probability that I did mainly because I got interested in a series of lectures he gave about Stein's method (more about that below) during my second year.

Ken: When did you realize that you wanted to become a mathematician? Do you remember a specific experience that got you excited about mathematics?

Elizabeth: I'd always sort of assumed I'd be an academic scientist of some sort, and when I started college I thought I'd probably be a physicist. But I found when taking classes, especially in my sophomore year when I started taking "real math" courses (that is, proof-based courses for math majors like abstract algebra and real analysis, as opposed to calculus, which is taught to a large audience of mostly non-math majors) that the way my mind worked seemed perfectly suited to being a mathematician. Math classes were certainly hard for me, but hard in a different way from other classes; math made sense to me in a way that physics didn't. I felt like I'd found my niche, and never really considered doing anything else.

Ken: What do you do, as a mathematician?

Elizabeth: That's a great question; I think most people really aren't clear on what mathematicians do all day. I once saw a great description of a mathematician's week: "Monday, tried to prove theorem. Tuesday, tried to prove theorem. Wednesday, tried to prove theorem. Thursday, tried to prove theorem. Friday: theorem false." So for one thing, I, like most mathematicians, am wrong a lot. But that doesn't really answer your question: what does it mean to spend your days and years trying to prove theorems? Sometimes it starts with a conjecture: something you read, a conversation with another mathematician, or something that you've been thinking about on your own makes you suspect that a specific statement is true. So you think about how to try to prove it: read other people's papers looking for ideas, scribble thoughts as they come to you, or just sit and think about it. Other times it's more exploratory: "I wonder what happens if you try to apply this argument in this other setting?" I usually try quite a few blind alleys, but can normally tell when I'm actually on to something. Of course, I don't know for sure if I've really proven something until I write it down carefully, as if I'm explaining it to something else. I really believe that you don't understand anything until you can explain it to someone else, and trying to do so is a great exercise to clarify your ideas and figure out which things you still don't understand.

Ken: You mentioned a mathematical tool that you like to use: Stein's method of exchangeable pairs. Can you explain to us what that is, or what it is essentially based on?

Elizabeth: Okay; I think most people are familiar with the bell-shaped curve, what I would call the Gaussian distribution. There's a general class of theorems called "central limit theorems", which are about showing that some specific quantity "looks Gaussian". The classic example is in terms of coin tossing. If you toss a coin 1,000 times, you expect to get about 500 heads and 500 tails. But of course you probably won't get *exactly* 500 of each, just close to that. So you could look a little closer and ask: if you perform the experiment millions of times and make a histogram of the number of heads from each experiment, what will it look like? We expect a big peak at 500, but can we say more about the shape of the histogram? The answer is yes: it's a bell-shaped curve, and that fact is the classical central limit theorem. You can write down a formula for the bell-shaped curve, and it has a lot of special properties; all of the approaches to proving central limit theorems take advantage of at least one of these special properties. Stein's method is probably the newest. It was invented in the late sixties by Charles Stein. That might sound like quite awhile ago, but when you think about the fact that people have known about the bell-shaped curve for more than two hundred years, saying much of anything really new and important about it is pretty amazing. The special property that Stein's method takes advantage of is the fact that the function which describes the bell-shaped curve is the only function there is that satisfies a certain differential equation; if some other curve (like a curve fitted to the coin-tossing histogram described above) "comes close" to satisfying the same differential equation, then it's close to the bell-shaped curve. This was Stein's fundamental idea, and a huge number of papers have been written extending and exploiting this idea. The exchangeable pairs part has to do with a way of actually implementing Stein's insight. It turns out that you can try to detect whether a curve governing a histogram like the coin-tossing one will satisfy this special differential equation by checking that the collection of outcomes of the experiment change in a very specific way when you change the experiment just a little. For example, you could take each of your coin-tossing experiments, but throw away the last toss and do it again, and replot the histogram. It will shift a little, and analyzing the particular way it shifts is a way to detect this differential equation at work. This all probably sounds a little silly, because if you already have the histogram, you can tell whether it looks bell-shaped or not, right? The thing is, often you don't have the histogram (say, because it's too hard to compute), but the amazing thing is that even if you can't plot the histogram, you can sometimes still figure out some things about how it would look if you could plot it, like how it would change if you tweaked the experiment a little.

Ken: Can you explain to us one of your own results?

Elizabeth: One of my favorite results is a central limit theorem in the context of Riemannian geometry. Riemannian geometry is about the geometry of curved spaces—think of the surface of a sphere as opposed to the surface of a table. On such a curved space, there are special functions (called "eigenfunctions of the Laplacian") which encode a lot of information about the geometry of the space—they are related to the famous question "Can you hear the shape of a drum?" In general, if you have a complicated function on a complicated space and you're trying to get a handle on what it's like, one interesting thing to think about is the distribution of the values it takes on; like the distribution of students' grades on an exam, knowing about which values the function takes on and how often might give you meaningful information. What I proved was a result about when the value distribution of one of these special functions looks like the bell-shaped curve. In particular, I showed that if you pick one of these functions on a high-dimensional sphere at random, it will probably have a value-distribution that looks like a bell-shaped curve. I like the mix of fields here, the interaction between geometry and probability.

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Another reason I like this result is that it is an example of something I think is really interesting: the study of "high-dimensional phenomena". Here, "high" means tending to infinity, so even a million dimensions isn't considered high. The idea of high dimensions is a little hard to wrap your head around; most of us are used to thinking in three dimensions, and that's it. But one of the great things about math is that once you turn your intuition about the world (in this case, about three-dimensional space) into a mathematical framework, you can abstract and generalize it without needing to follow any kind of rules that you feel hampered by from the physical world; the only rules you need to follow are mathematical ones. So you can start talking about space of arbitrarily many dimensions, and make sense of it.

To be continued…

An Interview with Elizabeth Meckes, Part II

Ken: Last time we were talking about high-dimensional spaces and phenomena in high-dimensions; can you tell us more about how they come up?

Elizabeth: For me, phenomena in high-dimensional spaces are interesting from a purely abstract mathematical perspective; that is, I just think they're interesting in their own right. But amazingly, sometimes the theories built up by mathematicians that come from this kind of generalization and abstraction, and are carried out purely because the mathematicians find them interesting, can turn out to be useful again back in the real world. This is what the famous physicist and mathematician Eugene Wigner called "the unreasonable effectiveness of mathematics in the natural sciences". In this case, it turns out that high-dimensional spaces are crucially important in applications all over the place: engineering, computing, physics, biology, chemistry, and more. You might think that as the dimension of a space goes up, things just get more and more complicated, and to some extent that's true. But it's also true that sometimes in high dimensions, patterns emerge from the chaos. The result I mentioned about value distributions of eigenfunctions on the sphere is like that; as the dimension goes up, these special functions are horrendously complicated to write down (you can do it, but trust me, you don't want to see the formulas). Even so, looked at in a certain way, that complication washes out and they all look pretty much the same, and pretty simple.

Ken: Fascinating! Do you have any advice for how best to learn mathematics?

Elizabeth: Yes, although I won't promise I've always followed my own advice (on the other hand, when I haven't I've usually regretted it).

First of all, learning math and doing math are really the same thing; it's an active process. There's a reason that math teachers assign so much homework: there's a real limit to how much understanding you can gain by watching someone else do math. To really understand something you have to work through it yourself. This is not in any way to suggest that teachers can't help; the insights that math teachers can provide about how to think about things can be incredibly valuable, but only in conjunction with struggling with it on your own. And I do mean struggling; understanding math doesn't come easy to anyone (if it has to you so far, don't worry— if you keep at it long enough you'll get to be as confused as the best of them).

Secondly, talking to other people about math is probably the best tool there is to gain better understanding. This includes people at all levels: those who know a lot more than you, a lot less than you, and everyone in between. Like I said before, explaining something to someone else is a fantastic way to clarify your own thinking and figure out which things you're still unsure about. When I think I've figured out something important (or just something that was hard for me and may not be all that important), I usually run it by someone else. This is frequently how I discover mistakes in my work. It helps the other way too; I can't count the number of times I've been stuck on something, went to ask someone else for help, and in the course of explaining the problem, I managed to figure out the solution.

Ken: To what extent is having female role models important for girls getting into math?

Elizabeth: This is a very personal issue, and my best advice on the subject is: Don't let anyone answer this question for you. Of course, it's great to have role models you can relate to (female or male), and on the other hand, you can excel without them. The girls who come to Girls' Angle are lucky in this respect, since they already have access to a lot of women who are top mathematics researchers.

The point I want to make is: when confronted with a choice that forces you to weigh how important it is to have a female mathematician as a role model, trust yourself. You are the best judge of how much this matters; if you feel that having a role-model or mentor is important to you, by all means seek one out. On the other hand, if the path you find most attractive doesn't involve such a mentor, then don't let that hold you back.

Ken: Do you have any hobbies aside from math?

Elizabeth: I must say I find this question a little strange, because I don't consider math a hobby; it's my job. This is actually an important point, and I'm glad to have the opportunity to discuss it. There's an idea out there that mathematicians as a rule are totally in love with mathematics and wouldn't stop to eat or sleep unless someone made them. This is really too bad, because I think that young people just getting started in math (particularly graduate students) can get really discouraged by this, and think that they shouldn't be mathematicians because that's not them. Anecdotally, this is particularly a problem for young women. Probably a lot of the girls that come to Girls' Angle do think of math as a hobby, and that's great. Some of them will continue in math and become mathematicians, which is also great. And the vast majority of those will probably some time, in graduate school if not sooner, stop thinking of math as a hobby and start thinking of it as a job. And that's fine, and probably close to inevitable, no matter what you decide to do with your life. There's a big difference between doing something for fun and doing it as a career, and anything you spend enough of your time on to call a career will be hard or boring or frustrating or all of the above, at least some of the time. It doesn't mean that it's not the right career for you; if you're lucky enough to be doing something for a living that you used to do for fun, it's probably one of the best career choices you could have made. But it will still be a drag sometimes, so it's good to have other things in your life too.

In light of all that, I'll rephrase the question as: "Other than math, how do you spend most of your time?"

Firstly, I have a wonderful family and spend most of my free time with my husband, Mark, and our two-year old daughter, Juliette. Mark is also a mathematician and people are always asking whether Juliette will be a mathematician, too. I always say, "I sure hope not". I want her to feel free to find her own thing (although I certainly wouldn't interfere if she did decide to do math). For now, she can almost count to ten, although for some reason she usually skips three. It's wonderful to be a mother.

One of the main activities I spend time on at home is cooking. There's a great moment that I really relate to in the movie "Julie and Julia", when Julia Child's husband asks her what she likes to do and she says with a laugh, "Eat!" All of my family like to eat well, and I spend a lot of my evenings and weekends in the kitchen. Juliette's favorite food is duck confit ravioli.

Ken: Do you have any advice for the girls that come to Girls' Angle?

Elizabeth: Just that I'm really impressed with the initiative of the girls that are getting involved in interesting math outside of school— keep it up! Sometimes it's a bumpy curve, but on average it just keeps getting better.

Ken: Thank you so much for this interview!

To really understand something you have to work through it yourself.